Chap2

2023年3月7日 23:50

1. 设 $X=i$ 为中 i 等奖

$X=0$ 为未中奖

$$P(X=1)=\frac{1}{C_{33}^{6}C_{16}^{1}}=\frac{1}{17721088}\approx 0.0000056\%$$

$$P(X=2)=\frac{15}{C_{33}^{6}C_{16}^{1}}=\frac{15}{17721088}\approx 0.0000846\%$$

$$P(X=3)=\frac{C_6^5C_{27}^1}{C_{33}^{6}C_{16}^{1}}=\frac{162}{17721088}\approx 0.00091\%$$

$$P(X=4)=\frac{C_6^5C_{27}^1\cdot 15+C_6^4C_{27}^2}{C_{33}^{6}C_{16}^{1}}=\frac{7695}{17721088}\approx 0.0434\%$$

$$P(X=5)=\frac{C_6^4C_{27}^2\cdot 15+C_6^3C_{27}^3}{C_{33}^{6}C_{16}^{1}}=\frac{137475}{17721088}\approx 0.7758\%$$

$$P(X=6)=\frac{C_6^2C_{27}^4+C_6^1C_{27}^5+C_{27}^6}{C_{33}^{6}C_{16}^{1}}=\frac{1043640}{17721088}\approx 5.88\%$$

$$P(X=0)=1-\_\_=93.29\%$$

(2) $P(\text{中奖})=1-P(X=0)=6.71\%$

$P(\text{一等奖})=P(X=1)+P(X=2)=0.0000902\%$

2. $P(X=1)=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\cdots\times\frac{98}{99}=\frac{1}{99}$

$P(X=2)=\underbrace{\frac{1}{2}\times\frac{1}{3}\times\frac{2}{4}\times\frac{3}{5}\times\cdots\times\frac{97}{99}}_{\text{第3球进}}+\underbrace{\frac{1}{2}\times\frac{1}{3}\times\frac{2}{4}\times\frac{3}{5}\times\cdots\times\frac{97}{99}}_{\text{第4球进}}$

$+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}\times\cdots\times\frac{97}{99}+\cdots+\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\cdots\times\frac{1}{99}$

第3球进　　　　　　　　　　　　第4球进

$$+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}\times\cdots\times\frac{97}{99}+\cdots+\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\cdots\times\frac{1}{99}$$

第5球进　　　　　　　　　　　　第100球进

$$=98\times\frac{1}{98\times 99}=\frac{1}{99}$$

...

$$P(X=99)=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\cdots\times\frac{98}{99}=\frac{1}{99}$$

分布律：$P(X=i)=\frac{1}{99}\quad(1\le i\le 99,\ i\in Z)$

3.

X	100	80	50	-60
$P(X)$	0.6	0.2	0.1	0.1

4. $P(X=10)=(1-0.2)^5=32.77\%$

$P(X=5)=C_5^1\cdot 0.2\cdot(1-0.2)^4=40.96\%$

$P(X=0)=C_5^2\, 0.2^2(1-0.2)^3=20.48\%$

$P(X=-2)=1-\_\_\ =5.79\%$

5. (1) $F(x)=\begin{cases}0, & x<-1\\ 0.25, & -1\le x<1\\ 0.75, & 1\le x<2\\ 1, & x\ge 2\end{cases}$

(2) $P(X\le 0)=F(0)=0.25$

$P(0.5<X\le 1.5)=F(1.5)-F(0.5)=0.5$

$P(1\le X\le 2)=F(2)-F(1^-)=0.75$

$P(1<X\le 2)=F(2)-F(1)=0.25$

6. $P(X=1)=\frac{8}{10}=0.8$

$P(X=2)=\frac{2}{10}\times\frac{8}{9}=0.178$

$P(X=3)=\frac{2}{10}\times\frac{1}{9}=0.022$

$P(X=2)=\frac{2}{10}\times\frac{8}{9}=0.178$

$P(X=3)=\frac{2}{10}\times\frac{1}{9}=0.022$

X	1	2	3
P	0.8	0.178	0.022

$$F(x)=\begin{cases}0, & x<1\\ 0.8, & 1\le x<2\\ 0.978, & 2\le x<3\\ 1, & x\ge 3\end{cases}$$

7. $P(X=k)=0.6^{k-1}\cdot 0.4$

$P(X为偶数)=0.6\cdot 0.4+0.6^3\cdot 0.4+\cdots$

$=\sum_{i=1}^{\infty} 0.24\cdot 0.36^{i-1}$

$=\frac{3}{8}$

$P(X>2)=1-P(X=1)-P(X=2)$

$=1-0.4-0.24=0.36$

8. $P(X=k)=C_{20}^{k}\,0.2^{k}\,0.8^{20-k}$

$P(X\ge 1)=1-P(X=0)=1-0.8^{20}=98.85\%$

X最有可能取值为 $np=4$

设X为A出现次数

9. (1) $P(X=0)=0.7^4=0.2401$

$P(X=1)=C_4^1\,0.3\cdot 0.7^3=0.4116$

$P(X\ge 2)=1-$ ____ $=0.3483$

$P(B)=0.4116\times 0.6+0.3483=59.53\%$

(2) $P(X=1\mid B)=\frac{P(B\cdot X=1)}{\ }=\frac{0.4116\times 0.6}{\ }=41.48\%$

(2) $P(X=1|B)=\frac{P(B\cdot X=1)}{P(B)}=\frac{0.416\times 0.6}{0.5953}=41.48\%$

10. $P_4=0.6^4=12.96\%$

$P_5=C_4^1\cdot 0.4\cdot 0.6^4=20.74\%$

$P_6=C_5^2\, 0.4^2\cdot 0.6^4=20.74\%$

$P_7=C_6^3\cdot 0.4^3\cdot 0.6^4=16.59\%$

甲胜概率 $=P_4+P_5+P_6+P_7=71.03\%$

三局两胜：$P_2=0.6^2=0.36$

$P_3=C_2^1\, 0.4\cdot 0.6^2=0.288$

甲胜概率 $=P_2+P_3=64.8\%$

故七局四胜更有利

11. $P(X=-1)=(\frac{5}{6})^3=\frac{125}{216}$

$P(X=1)=C_3^1(\frac{1}{6})\cdot(\frac{5}{6})^2=\frac{75}{216}$

$P(X=2)=C_3^2(\frac{1}{6})^2\frac{5}{6}=\frac{15}{216}$

$P(X=3)=(\frac{1}{6})^3=\frac{1}{216}$

?

12. $Y\sim P(\lambda p)$

$Z\sim P(\lambda(1-p))$

不相互独立

13. $P(X=0)=0.999^{1000}$ $\quad \lambda=np=1000\times 0.001=1$

$\approx\frac{\lambda^0}{0!}e^{-\lambda}$

$=e^{-1}=36.79\%$

14. $P(X\geq 2)=1-P(X=0)-P(X=1)$ $\quad \lambda=np=8$

14. $P(X\geq 2)=1-P(X=0)-P(X=1)$ $\lambda=np=8$

$\approx 1-e^{-8}-8\cdot e^{-8}$

$=99.7\%$

15. (1) $\lambda_1=0.512$

$P(X\geq 1)=1-P(X=0)$

$\approx 1-e^{-0.512}=40.1\%$

(2) $\lambda_2=5.12$

$P(X\geq 1)=1-P(X=0)$

$\approx 1-e^{-5.12}=99.4\%$

16. 设X为退票人数 ，$X\sim B(52,0.05)$

$np=2.6$

$P=P(X=0)+P(X=1)$

$\approx e^{-2.6}+2.6\cdot e^{-2.6}$

$=27\%$

17. (1) 设X为中奖注数 $\lambda=0.01$

用二项分布近似 $X\sim B(100,0.0001)$

中奖：$1-P(X=0)\approx 1-e^{-0.01}$

$=0.995\%$

(2) 设买n注，$\lambda=0.0001n$

$1-P(X=0)\approx 1-e^{-0.0001n}>0.95$

$e^{-0.0001n}<0.05$

$-0.0001n<\ln 0.05$

$n>\frac{\ln 0.05}{-0.0001}=29957.32$

取整，至少买 29958注

取整,至少买 29958注

18. (1) $P(X=1)=P(X\le 1)-P(X<1)=F(1)-F(1^-)=\frac{1}{4}$

$P(X=2)=P(X\le 2)-P(X<2)=F(2)-F(2^-)=\frac{1}{2}$

$P(X=3)=P(X\le 3)-P(X<3)=F(3)-F(3^-)=\frac{1}{6}$

(2) $P(\frac{1}{2}<x<\frac{3}{2})=P(X<\frac{3}{2})-P(X<\frac{1}{2})=F(\frac{3}{2}^-)-F(\frac{1}{2}^-)=\frac{1}{2}$

19. $P(X=1)=P(X\le 1)-P(X<1)$

$=F(1)-F(1^-)$

$=1-(a+b)=\frac{1}{4}$

$\Rightarrow a+b=\frac{3}{4}$

$F(-1^+)=F(-1)$

$\Rightarrow b-a=\frac{1}{8}$

$\therefore b=\frac{7}{16},\ a=\frac{5}{16}$

20. $\int_{-\infty}^{+\infty} f(x)dx=\frac{3}{2}a+b=1$

$\int_1^2 ax\,dx=\int_2^3 b\,dx \Rightarrow \frac{3}{2}a=b$

$\therefore \begin{cases} a=\frac{1}{3} \\ b=\frac{1}{2} \end{cases}$

21. (1) $\int_{-\infty}^{+\infty} f(x)dx= a\cdot \arctan x\Big|_{-\infty}^{+\infty}$

$=\pi a=1$

$\therefore a=\frac{1}{\pi}$

(2) $F(x)=\int_{-\infty}^{x} f(t)dt=\int_{-\infty}^{x}\frac{1}{\pi}\cdot\frac{1}{1+t^2}dt$

$=\frac{1}{\pi}\cdot \arctan t\Big|_{-\infty}^{x}$

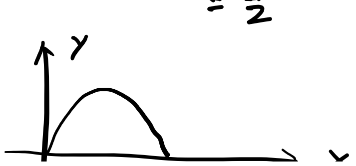
$=\frac{\arctan x+\frac{\pi}{2}}{\pi}=\frac{\arctan x}{\pi}+\frac{1}{2}$

(3) $P(|X|<1)=P(-1<X<1)$

$=F(1)-F(-1)$

$=\frac{1}{2}$

22.



22.

$$F(x)=\frac{\int_0^x (2t-t^2)dt}{\int_0^2 (2x-x^2)dx} \quad (0\le x\le 2)$$

$$=\frac{3}{4}x^2-\frac{1}{4}x^3$$

$$\therefore F(x)=\begin{cases}\frac{3}{4}x^2-\frac{1}{4}x^3, & 0\le x\le 2\\ 0, & \text{other}\end{cases}$$

23. (1) $F(1)=F(1^-)$

$b+1=0 \Rightarrow b=-1$

$F(e)=F(e^+)$

$ae^2-e^2+1=1 \Rightarrow a=1$

$\therefore a=1, b=-1$

(2) $1\le x\le e$时

$f(x)=F'(x)=2x\ln x - x$

$$\therefore f(x)=\begin{cases}2x\ln x-x, & 1\le x\le e\\ 0, & \text{other}\end{cases}$$

24. $$f(x)=\begin{cases}\frac{1}{10}, & -5<x<5\\ 0, & \text{other}\end{cases}$$

$$F(x)=\begin{cases}0, & x\le -5\\ \frac{x+5}{10}, & -5<x<5\\ 1, & x\ge 5\end{cases}$$

$\Delta = X^2-4\ge 0 \quad \Rightarrow X\le -2\ /\ X\ge 2$

$Ans = P(X\le -2)+P(X\ge 2)$

$= F(-2)+1-F(2)$

$= 0.3+1-0.7 = 0.6$

25. $$f(x)=\begin{cases}\frac{1}{30}, & 0<x<30\\ 0, & \text{other}\end{cases}$$

$Ans = P(10<X<15)+P(25<X<30)$

$= \int_{10}^{15} f(x)dx + \int_{25}^{30} f(x)dx$

$= \frac{1}{3}$

26. $P(X>2) = \frac{2}{3}$

$Ans = C_3^2 (\frac{2}{3})^2 \frac{1}{3} + (\frac{2}{3})^3$

$= \frac{20}{27}$

27. 取 $a=0, b=x$

$F(x) = \int_0^x f(t)dt = F(x) - F(0)$

$= k(x-0)$

$= kx$

$\therefore f(x) = F'(x) = k$ 为常数

$\therefore X \sim U(0,1)$

28. $f(x) = \begin{cases} 0, & x<0 \\ e^{-x}, & x \geq 0 \end{cases}$

$F(x) = \begin{cases} 0, & x<0 \\ 1-e^{-x}, & x \geq 0 \end{cases}$

(1) $P(X>2) = 1 - P(X \leq 2)$

$= 1 - F(2)$

$= 1 - (1-e^{-2}) = e^{-2} \approx 13.53\%$

(2) $P(X>2+2 \mid X>2) = P(X>2) = 13.53\%$

29. $F(x) = \begin{cases} 0, & x<0 \\ 1-e^{-\frac{1}{5}x}, & x \geq 0 \end{cases}$

$P(X>10) = 1 - (1-e^{-2}) = e^{-2}$

$Ans = 1 - (1-e^{-2})^5 \approx 51.67\%$

30. $\Rightarrow$

$P(X \leq t+x \mid X>t) = \frac{[1-e^{-\lambda(t+x)}] - [1-e^{-\lambda t}]}{1-(1-e^{-\lambda t})}$

$= \frac{e^{-\lambda t} - e^{-\lambda t} \cdot e^{-\lambda x}}{e^{-\lambda t}}$

$= 1 - e^{-\lambda x}$

$= P(X \leq x)$

$\Leftarrow$: $P(X \leq t+x \mid X>t) = \frac{F(t+x) - F(t)}{1-F(t)}$

$P(X \leq x) = F(x)$

$F(t+x)-F(t)=(1-F(t))F(x)$

$F(t+x)=F(x)+F(t)-F(x)F(t)$

$1-F(t+x)=1-F(x)-F(t)+F(x)F(t)$

$=[1-F(x)][1-F(t)]$

$\ln[1-F(t+x)]=\ln[1-F(x)]+\ln[1-F(t)]$

令 $g(x)=\ln(1-F(x))$

则 $g(t+x)=g(x)+g(t)$，$t,x>0$

$\therefore g(x)=g(1)x$

$\because g(1)<0$，记 $\lambda=-g(1)>0$

$g(x)=-\lambda x=\ln(1-F(x))$

$\therefore F(x)=1-e^{-\lambda x}$

$\therefore X$ 服从指数分布

(2) $P(X=k)=q^{k-1}p$ $(p+q=1)$

$P(X\le k)=\sum_{i=1}^{k}q^{i-1}p$

$=\frac{p(1-q^k)}{1-q}=1-q^k$

剩余证明与(1)相似

31. (1) $\mu=1$, $\sigma=2$

$P(0\le X\le 4)=P\left(\frac{0-1}{2}\le\frac{X-1}{2}\le\frac{4-1}{2}\right)$

$=P\left(-\frac{1}{2}\le\frac{X-1}{2}\le\frac{3}{2}\right)$

$=\Phi\left(\frac{3}{2}\right)-\Phi\left(-\frac{1}{2}\right)$

$=\Phi\left(\frac{3}{2}\right)-\left(1-\Phi\left(\frac{1}{2}\right)\right)$

$=0.933+0.691-1$

$=0.624$

$P(X>2.4)=P\left(\frac{X-1}{2}\ge 0.7\right)$

$=1-\Phi(0.7)$

$=0.242$

$P(|X|>2)=P(X<-2)+P(X>2)$

$=P\left(\frac{X-1}{2}<-1.5\right)+P\left(\frac{X-1}{2}>0.5\right)$

$=\Phi(-1.5)+(1-\Phi(0.5))$

$=1-\Phi(1.5)+1-\Phi(0.5)$

$=2-0.933-0.691=0.376$

(2) $P(X>c)=2P(X\le c)$

$1-P(X\le C)=2P(X\le C)$

$P(X\le C)=\frac{1}{3}$

$P(\frac{X-1}{2}\le\frac{C-1}{2})=\Phi(\frac{C-1}{2})=1-\Phi(\frac{1-C}{2})=\frac{1}{3}$

$\Phi(\frac{1-C}{2})=\frac{2}{3}$

$\frac{1-C}{2}=0.43$

$C=0.14$

32. (1) $P=\frac{8}{10}=0.8$

(2) $\mu=100,\ \sigma=2$

$P(96<X<104)=P(-2<\frac{X-100}{2}<2)$
$=\Phi(2)-\Phi(-2)$
$=2\Phi(2)-1$
$=0.954$

33. 选市区 $\mu=30,\ \sigma=10$

$P(X<50)=P(\frac{X-30}{10}<2)$
$=\Phi(2)$

$P(X<45)=P(\frac{X-30}{10}<1.5)=\Phi(1.5)$

选环城公路 $\mu=40,\ \sigma=4$

$P(X<50)=P(\frac{X-40}{4}<2.5)=\Phi(2.5)$

$P(X<45)=P(\frac{X-40}{4}<1.25)=\Phi(1.25)$

(1) 选市区, (2) 选环城

34.

X	2	3	4	5	6	7	8	9	10	11	12
P	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

35.

X	1	2	3	4	5	6
P	$\frac{1}{216}$	$\frac{7}{216}$	$\frac{19}{216}$	$\frac{37}{216}$	$\frac{81}{216}$	$\frac{91}{216}$

36.

Y_1	-3	-1	1	3
P	0.4	0.1	0.3	0.2

Y_2	0	1	2
P	0.3	0.3	0.4

Y_3	0	1	4
P	0.1	0.7	0.2

37. (1) $F(-\infty)=-\frac{\pi}{2}b+a=0$

$F(+\infty)=\frac{\pi}{2}b+a=1$

$a=\frac{1}{2},\ b=\frac{1}{\pi}$

(2) $f_X(x)=\frac{1}{\pi(x^2+1)}$

$$P(Y=y)=P(3-\sqrt[3]{x}=y)$$
$$=P(x=(3-y)^3)$$
$$=|f_x[(3-y)^3]\,d((3-y)^3)|$$
$$=\frac{1}{\pi[(y-3)^6+1]}\cdot 3\cdot(3-y)^2\,dy$$

$$\therefore p(y)=\frac{3(y-3)^2}{\pi[(y-3)^6+1]}$$

(3) 令 $Y=\frac{1}{X}$

$$P(Y=y)=P(\frac{1}{X}=y)=P(X=\frac{1}{y})$$
$$=|f_X(\frac{1}{y})\,d\frac{1}{y}|$$
$$=\frac{1}{\pi(\frac{1+y^2}{y^2})}\cdot\frac{1}{y^2}\,dy$$
$$=\frac{1}{\pi(1+y^2)}\,dy$$

$$\therefore f_Y(y)=\frac{1}{\pi(1+y^2)}$$

$\therefore X$ 与 $\frac{1}{X}$ 分布相同

38. 设 $V\sim N(\mu,\sigma)$，$f(v)=\frac{1}{\sqrt{2\pi}\sigma}\exp\{\frac{(x-\mu)^2}{-2\sigma^2}\}$

$$E=\frac{1}{2}mV^2$$

$$P(E=e)=P(\frac{1}{2}mV^2=e)$$
$$=P(V=\pm\sqrt{\frac{2e}{m}})$$
$$=\frac{1}{\sqrt{2\pi}\sigma}\exp\{\frac{(-\sqrt{\frac{2e}{m}}-\mu)^2}{-2\sigma^2}\}\cdot\frac{1}{\sqrt{2em}}\,de$$
$$+\frac{1}{\sqrt{2\pi}\sigma}\exp\{\frac{(\sqrt{\frac{2e}{m}}-\mu)^2}{-2\sigma^2}\}\cdot\frac{1}{\sqrt{2em}}\,de$$

$$\therefore f_{E_k}(e)=\frac{1}{2\sigma\sqrt{\pi em}}\left[\exp\{\frac{(-\sqrt{\frac{2e}{m}}-\mu)^2}{-2\sigma^2}\}+\exp\{\frac{(\sqrt{\frac{2e}{m}}-\mu)^2}{-2\sigma^2}\}\right]$$

39. if $t\le 0$, $Y=X$.

if $t>0$, $Y=\begin{cases}X, & X>t\\ 0, & X\le t\end{cases}$

$$\therefore f(y)=\begin{cases}\lambda e^{-\lambda y}, & y>t\\ 0, & y\le t\end{cases}$$

40. $f(x)=\begin{cases}1, & 0<x<1\\ 0, & \text{other}\end{cases}$

(1) $P(Y_1=y)=P(e^X=y)$
$$=P(X=\ln y)$$
$$=f(\ln y)\,\frac{1}{y}\,dy$$

$f_{Y_1}(y)=\begin{cases}\frac{1}{y}, & 1<y<e\end{cases}$

0, other

(2) $P(Y_2=y)=P(X^{-1}=y)$

$=P(X=\frac{1}{y})$

$=f(\frac{1}{y})\frac{1}{y^2}dy$

$f_{Y_2}(y)=\begin{cases}\frac{1}{y^2}, & x>1\\ 0, & x\le 1\end{cases}$

(3) $P(Y_3=y)=P(-\frac{1}{\lambda}\ln X=y)$

$=P(X=e^{-\lambda y})$

$=f(e^{-\lambda y})\lambda e^{-\lambda y}dy$

$\therefore f_{Y_3}(y)=\begin{cases}\lambda e^{-\lambda y}, & y>0\\ 0, & y\le 0\end{cases}$

41、 $f(x)=\begin{cases}\frac{1}{\pi}, & -\frac{\pi}{2}<x<\frac{\pi}{2}\\ 0, & \text{Other}\end{cases}$

$P(Y_1=y)=P(\tan X=y)$

$=P(X=\arctan y)$

$=f(\arctan y)\cdot\frac{1}{y^2+1}dy$

$f_{Y_1}(y)=\frac{1}{\pi(y^2+1)}$

$P(Y_2=y)=P(\cos X=y)$

$=P(X=\arccos y)$

$=f(\arccos y)\cdot\frac{1}{\sqrt{1-y^2}}dy$

$f_{Y_2}(y)=\begin{cases}\frac{1}{\pi\sqrt{1-y^2}}, & 0<y\le 1\\ 0, & \text{Other}\end{cases}$

42、 $Y\in(0,1)$，故当$y\in(0,1)$时

$P(Y\le y)=P(F(X)\le y)=P(X\le F^{-1}(y))$

$=F(F^{-1}(y))$

$=y$

$\therefore Y\sim U(0,1)$

43、 $f(x)=\begin{cases}e^{-x}, & x\ge 0\\ 0, & x<0\end{cases}$

$P(Y_1=y)=P(X^2=y)$

$=P(X=\sqrt{y})$

$= f(\sqrt{y})\frac{1}{2\sqrt{y}}dy$

$f_{Y_1}(y)=\begin{cases}\frac{e^{-\sqrt{y}}}{2\sqrt{y}}, & y>0\\ 0, & y\le 0\end{cases}$

$P(Y_2=y)=P(1-e^{-X}=y)$

$=P(X=\ln\frac{1}{1-y})$

$=f(\ln\frac{1}{1-y})\cdot\frac{1}{1-y}dy$

$=\begin{cases}1, & 0<y<1\\ 0, & other\end{cases}$

44. $P(Y=y)$ 令$Y=g(X)$

$=P(g(X)=y)$

$=P(X=g^{-1}(y))$

令$g^{-1}(y)=\varphi(y)$

$=P(X=\varphi(y))$

$=f(\varphi(y))\varphi'(y)dy$

$\therefore f_Y(y)=2(1-\varphi(y))\varphi'(y)=e^{-y}$，且$\varphi(0)=0$

瞎gb乱解得，$x=\varphi(y)=1-e^{-\frac{1}{2}y}$

$\therefore y=g(x)=-2\ln(1-x)$

45. $P(Y=n)=\int_n^{n+1}\lambda e^{-\lambda x}dx$

$=e^{-\lambda n}(1-e^{-\lambda})$

Y服从参数为$1-e^{-\lambda}$的几何分布.

$P(Z\le z|Y=n)=\frac{\int_n^{n+z}\lambda e^{-\lambda x}dx}{e^{-\lambda n}(1-e^{-\lambda})}=\frac{1-e^{-\lambda z}}{1-e^{-\lambda}}$

$\therefore$ Y,Z相互独立

$f(z)=\frac{\lambda e^{-\lambda z}}{1-e^{-\lambda}}$，$0<z<1$

46. $y\ge 1$时 $p(y)=\lambda e^{-\lambda y}$

$0<x<1\Rightarrow -1<y<0$

$P(Y=y)=P(-X^2=y)$

$=P(X=\sqrt{-y})$

$=f(\sqrt{-y})\frac{1}{\quad}dy$

$$\frac{}{2\sqrt{-y}}\ )$$

$$p(y)=\lambda e^{-\lambda\sqrt{-y}}\frac{1}{2\sqrt{-y}}$$

$$\therefore p(y)=\begin{cases}\lambda e^{-\lambda y}, & y\geq 1\\ \dfrac{\lambda e^{-\lambda\sqrt{-y}}}{2\sqrt{-y}}, & -1<y<0\end{cases}$$

47. $X\sim N(0,1)$

(1) $P(Y_1=y)=P(e^X=y)$

$$=P(X=\ln y)$$

$$=f(\ln y)\frac{1}{y}dy$$

$$f_{Y_1}(y)=\frac{1}{\sqrt{2\pi}y}\exp\{-\frac{(\ln y)^2}{2}\}\quad ,y>0$$

(2) $P(Y_2=y)=P(|X|=y)$

$$=P(X=-y)+P(X=y)$$

$$=f(-y)dy+f(y)dy$$

$$=[f(-y)+f(y)]dy$$

$$f_{Y_2}(y)=\sqrt{\frac{2}{\pi}}\exp\{-\frac{y^2}{2}\}\quad ,y\geq 0$$

(3) $P(Y_3=y)=P(2X^2+1=y)$

$$=P(X=\pm\sqrt{\frac{y-1}{2}})$$

$$=f(\sqrt{\frac{y-1}{2}})\frac{1}{2\sqrt{2(y-1)}}dy$$

$$+f(-\sqrt{\frac{y-1}{2}})\frac{1}{2\sqrt{2(y-1)}}dy$$

$$=\frac{1}{2\sqrt{\pi(y-1)}}\exp(-\frac{y-1}{4})\quad ,y>1$$

48. $\int_{-\infty}^{+\infty}f(x)dx=\frac{9}{a}=1$

$$\therefore a=9$$

$$\therefore f(x)=\frac{1}{9}x^2$$

$1<y<2$时

$$F_Y(y)=P(Y=1)+P(1<Y\leq y)$$

$$=P(X>2)+P(1<X\leq y)$$

$$=\int_2^3 f(x)dx+\int_1^y f(x)dx$$

$$=\frac{y^3+18}{27}$$

$$\therefore F_Y(y)=\begin{cases}0, & y<1\\ \dfrac{y^3+18}{27}, & 1\leq y\leq 2\\ 1 & y>2\end{cases}$$

(2) $P(X\le Y) = P(X\le Y \mid X\le 1)\,P(X\le 1)$

$+P(X\le Y \mid 1<X<2)\,P(1<X<2)$

$+P(X\le Y \mid X\ge 2)\,P(X\ge 2)$

$= 1\times\int_0^1 f(x)\,dx + 1\times\int_1^2 f(x)\,dx + 0$

$=\int_0^2 f(x)\,dx$

$= \dfrac{8}{27}$